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Description and Smoothing of NC Motion Path Based on the Cubic Trigonometric Interpolation spline

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Abstract. In order to better describe the complex motion path of NC machining and realize the smooth transition between path segments, a kind of cubic trigonometric interpolation spline curve was put forward based on a set of special basis function. The spline curve which with adjustable shape satisfies the C^1 continuity and it can accurately describe some common engineering curves such as straight line, circular arc and free curve. According to the given information of control points, different shapes of interpolation spline curve can be gotten by changing the adjustment coefficients. Through selecting proper control points and shape adjustment coefficients near the corner, insert the spline curve can realize the smooth transition at the corner of adjacent NC motion path segments, which can ensure the stability of motion path and the continuous of feed speed. Meanwhile, it also can reduce the impact to NC machine.

Introduction

High-speed NC machining is the important way to improve efficiency and quality of parts machining. During the high-speed operation process of machine tool, it needs to ensure the stability of machine tool movements to avoid generating larger impacts which will affect the quality of parts machining, meanwhile to protect the machine tool feeding system [1]. In actual machining process, NC motion paths often consist of a number of straight lines and circular arcs. At present, to deal with the speed at the corner of adjacent NC machining path segments, the main method is to slow down the speed to zero at the end of the current processing segment, and then accelerate to command speed in the starting position of the next processing segment. Using this way to get through each corner with zero speed can avoid larger impacts to NC machine. However, this way will cause frequent start and stop of speed during the machining process, and it will seriously affect the improvement of parts machining efficiency [2]. Therefore, it needs to study new motion control method at the corner, which can make the transfer speed does not drop to zero, and reach the purpose of realizing the high-speed smooth transition between two adjacent path segments. Thus to improve the machining efficiency and limit the impact loads.

In dealing with the speed at the corner of adjacent machining path segments, literatures [3,4] proposed to add circular arc or quadratic curve for corner transition, in order to make the speed does not drop to zero. To a certain extent, this method improved the machining efficiency and reduced the impacts to machine tools. But it lacks the control of acceleration and the error control is not strict; when the corner is larger, the improvement of transfer speed will be limited, so this method does not well meet the needs of high-speed machining. Literature [5] put forward a speed control method with look-ahead; this method limited the impacts formed by the change of velocity vector at the corner; through the limited speed dropping at the corner to pursue the maximum machining efficiency. But it needs a large amount of pre-computation and it requires higher numerical control system. Literature [6] proposed a vector method to realize the smooth transition between two path segments. Through the anticipatory analysis of the motion parameters, this method can improve the transfer speed when the corner is larger; but when the corner is smaller, the transfer speed is still not smooth enough.

On the basis of cubic Hermite interpolation function[7,8], this paper puts forward a kind of adjustable shape cubic trigonometric polynomial interpolation spline curve, which can be used to describe the NC machining paths, such as straight line, circular arc and free curve segments, and it has properties of simple calculation, flexible structure, etc. The spline can also be used to describe the smooth transition curves at the corner and realize the high-speed smooth transition between NC machining path segments, which meets the needs of modern numerical control system for high-speed, stability and flexibility.

Cubic Trigonometric Interpolation Spline Curve

Basis Function of Spline. For any given value of variables λ and k, parameter u satisfies $0 \le u \le 1$; the following formula is called a set of cubic trigonometric spline basis function.

$$
\begin{cases}
B_{0,3}(u) = (1-\frac{2}{\pi}\lambda)C^3 + (\frac{2}{\pi}-\lambda)S^3 + \lambda(1-\frac{2}{\pi})S^2 - \frac{2}{\pi}S + \frac{2}{\pi}\lambda \\
B_{1,3}(u) = \left[\frac{2}{\pi}(k+1)\lambda - (1+\frac{2}{\pi})\right]C^3 + \left[\lambda(1+\frac{2}{\pi}) - \frac{2}{\pi}(k+1)\right]S^3 + \lambda(\frac{2}{\pi}k-1)S^2 + \frac{2}{\pi}C + \frac{2}{\pi}(k+1)S - \frac{2}{\pi}(k+1)\lambda + 1 \\
B_{2,3}(u) = \left[\frac{2}{\pi}(k+1) - \frac{2}{\pi}k\lambda\right]C^3 + \left[\frac{2}{\pi}k - \frac{2}{\pi}(k+1)\lambda\right]S^3 + \frac{2}{\pi}\lambda S^2 - \frac{2}{\pi}(k+1)C - \frac{2}{\pi}kS + \frac{2}{\pi}k\lambda \\
B_{3,3}(u) = -\frac{2}{\pi}kC^3 + \frac{2}{\pi}k\lambda S^3 - \frac{2}{\pi}k\lambda S^2 + \frac{2}{\pi}kC
\end{cases}
\tag{1}
$$

In formula: S is stand for $\sin\left(\pi u/2\right)$, C is stand for $\cos\left(\pi u/2\right)$.

Interpolation Spline Curve. Suppose that the group of given control points are q_i ($i = 0,1,\cdots, n$), then the spline curve segments can be defined as:

$$P_i(u) = \sum_{j=0}^{3} q_{i+j} B_{j,3}(u), \; 0 \le u \le 1, \; i = 0,1,\cdots, n-3. \tag{2}$$

The curve $P(u)$ which is composed of all small curve segments $P_i(u)$ is called the cubic trigonometric interpolation spline curve.

From expression (2), it is easy to know that the whole curve $P(u)$ formed by $n-2$ segments of small curves. To number i segment curve $P_i(u)$ exists:

$$\begin{aligned} &P_i(0) = q_i, P_i'(0) = (q_{i+1} - q_i) - k(q_{i+2} - q_{i+1}) \\ &P_i(1) = q_{i+1}, P_i'(1) = (q_{i+2} - q_{i+1}) - k(q_{i+3} - q_{i+2}) \end{aligned}. \tag{3}$$

And to number $i+1$ segment curve $P_{i+1}(u)$ exists:

$$\begin{aligned} &P_{i+1}(0) = q_{i+1}, P_{i+1}'(0) = (q_{i+2} - q_{i+1}) - k(q_{i+3} - q_{i+2}) \\ &P_{i+1}(1) = q_{i+2}, P_{i+1}'(1) = (q_{i+3} - q_{i+2}) - k(q_{i+4} - q_{i+3}) \end{aligned}. \tag{4}$$

Comparing formula (3) with (4), it can be obviously found that the adjacent curves $P_i(u)$ and $P_{i+1}(u)$ have the following connections:

$$\begin{aligned} &P_i(1) = P_{i+1}(0) = q_{i+1} \\ &P_i'(1) = P_{i+1}'(0) = (q_{i+2} - q_{i+1}) - k(q_{i+3} - q_{i+2}) \end{aligned}. \tag{5}$$

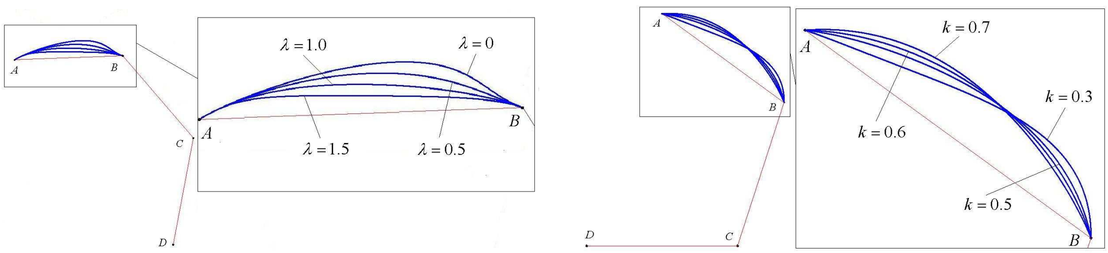
According to formulas (3), (4) and (5), a theorem can be gotten, that is the spline curve $P(u)$ interpolates the group of given control points from q_0 to q_{n-2} and it also satisfies C^1 continuity.

Spread out the spline curve expression (2) into a polynomial form, then can get:

$$\begin{aligned}\boldsymbol{P}(u)=\boldsymbol{q}_i&\left[(1-\frac{2}{\pi}\lambda)C^3+(\frac{2}{\pi}-\lambda)S^3+\lambda(1-\frac{2}{\pi})S^2-\frac{2}{\pi}S+\frac{2}{\pi}\lambda\right]+\\ \boldsymbol{q}_{i+1}&\left\{\left[\frac{2}{\pi}(k+1)\lambda-(1+\frac{2}{\pi})\right]C^3+\left[\lambda(1+\frac{2}{\pi})-\frac{2}{\pi}(k+1)\right]S^3+\lambda(\frac{2}{\pi}k-1)S^2+\frac{2}{\pi}C+\frac{2}{\pi}(k+1)S-\frac{2}{\pi}(k+1)\lambda+1\right\}+\\ \boldsymbol{q}_{i+2}&\left\{\left[\frac{2}{\pi}(k+1)-\frac{2}{\pi}k\lambda\right]C^3+\left[\frac{2}{\pi}k-\frac{2}{\pi}(k+1)\lambda\right]S^3+\frac{2}{\pi}\lambda S^2-\frac{2}{\pi}(k+1)C-\frac{2}{\pi}kS+\frac{2}{\pi}k\lambda\right\}+\\ \boldsymbol{q}_{i+3}&\left(-\frac{2}{\pi}kC^3+\frac{2}{\pi}k\lambda S^3-\frac{2}{\pi}k\lambda S^2+\frac{2}{\pi}kC\right)\end{aligned}; \qquad (6)$$

$$\begin{cases}X=x_i\left[(1-\frac{2}{\pi}\lambda)C^3+(\frac{2}{\pi}-\lambda)S^3+\lambda(1-\frac{2}{\pi})S^2-\frac{2}{\pi}S+\frac{2}{\pi}\lambda\right]+\\ \quad x_{i+1}\left\{\left[\frac{2}{\pi}(k+1)\lambda-(1+\frac{2}{\pi})\right]C^3+\left[\lambda(1+\frac{2}{\pi})-\frac{2}{\pi}(k+1)\right]S^3+\lambda(\frac{2}{\pi}k-1)S^2+\frac{2}{\pi}C+\frac{2}{\pi}(k+1)S-\frac{2}{\pi}(k+1)\lambda+1\right\}+\\ \quad x_{i+2}\left\{\left[\frac{2}{\pi}(k+1)-\frac{2}{\pi}k\lambda\right]C^3+\left[\frac{2}{\pi}k-\frac{2}{\pi}(k+1)\lambda\right]S^3+\frac{2}{\pi}\lambda S^2-\frac{2}{\pi}(k+1)C-\frac{2}{\pi}kS+\frac{2}{\pi}k\lambda\right\}+\\ \quad x_{i+3}\left(-\frac{2}{\pi}kC^3+\frac{2}{\pi}k\lambda S^3-\frac{2}{\pi}k\lambda S^2+\frac{2}{\pi}kC\right)\\ Y=y_i\left[(1-\frac{2}{\pi}\lambda)C^3+(\frac{2}{\pi}-\lambda)S^3+\lambda(1-\frac{2}{\pi})S^2-\frac{2}{\pi}S+\frac{2}{\pi}\lambda\right]+\\ \quad y_{i+1}\left\{\left[\frac{2}{\pi}(k+1)\lambda-(1+\frac{2}{\pi})\right]C^3+\left[\lambda(1+\frac{2}{\pi})-\frac{2}{\pi}(k+1)\right]S^3+\lambda(\frac{2}{\pi}k-1)S^2+\frac{2}{\pi}C+\frac{2}{\pi}(k+1)S-\frac{2}{\pi}(k+1)\lambda+1\right\}+\\ \quad y_{i+2}\left\{\left[\frac{2}{\pi}(k+1)-\frac{2}{\pi}k\lambda\right]C^3+\left[\frac{2}{\pi}k-\frac{2}{\pi}(k+1)\lambda\right]S^3+\frac{2}{\pi}\lambda S^2-\frac{2}{\pi}(k+1)C-\frac{2}{\pi}kS+\frac{2}{\pi}k\lambda\right\}+\\ \quad y_{i+3}\left(-\frac{2}{\pi}kC^3+\frac{2}{\pi}k\lambda S^3-\frac{2}{\pi}k\lambda S^2+\frac{2}{\pi}kC\right)\end{cases}. \qquad (7)$$

In formulas (7) and (8): S is stand for $\sin\left(\pi u/2\right)$, C is stand for $\cos\left(\pi u/2\right)$, $0\le u\le 1$.



(a) (b)

Fig. 1 Adjustable shape interpolation spline curve

From above can know, any four given control points completely determine a segment of adjustable shape cubic trigonometric interpolation spline curve. The curve interpolates the first two control points, and the latter two control points are used to calculate the endpoint slopes of the curve segment. Setting q_i, q_{i+1}, q_{i+2}, q_{i+3} respectively as A, B, C, D four assured points, adopting AutoLISP programming in the AutoCAD can draw a segment of adjustable shape cubic trigonometric interpolation spline curve, as shown in Fig. 1. Changing the value of λ and k can realize the shape adjustment of the curve segment. The four curves in Fig. 1(a) respectively corresponding to the spline curve segments when k takes a certain value of 0.5, $\lambda=0$, 0.5, 1 and 1.5; while the four curves in Fig. 1(b) respectively corresponding to the spline curve segments when λ takes a certain value of 1, $k=0.3$, 0.5, 0.6 and 0.7.

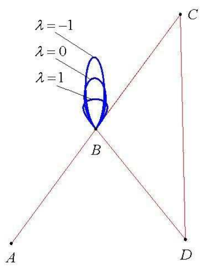


Fig. 2 Ring interpolation spline

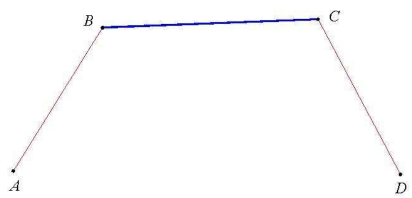


Fig. 3 Straight line interpolation spline

Furthermore, ring form interpolation spline curve can be generated when the control points are reused. As shown in Fig. 2, when the control points sequence is (B, B, C, D), i.e. point B is reused, then through programming can generate the ring spline curve segments. The three ring curves in Fig.

2 respectively corresponding to the spline curve segments when k takes a certain value of -1, $\lambda=-1$, 0, and 1. Keeping the three points A, B, C locate in one straight line, then the starting point vector of the ring spline is in the same direction with AB, and the end point vector is in the same direction with BD. This feature can be well used to deal with the corner smooth transition between adjacent path segments in NC machining.

Engineering Application of the Cubic Trigonometric Interpolation Spline Curve

Compared with the traditional Ferguson curve, Bezier curve and B-spline curve, the adjustable shape cubic trigonometric interpolation spline curve has better properties. It not only has the properties of interpolation and shape adjustable, but also because the expression of the spline curve contains trigonometric polynomial, by choosing proper control points and shape adjustment coefficients, the spline curve can accurately describe the common engineering curves such as straight line, circular arc and free curve, etc. Therefore, the spline curve can be used to describe the complex NC motion paths; meanwhile, through selecting proper control points near the corner of adjacent machining path segments and shape adjustment coefficients, it can generate the ring form or circular arc spline curve which can realize the high-speed smooth transition around the corner.

Straight Line Description of NC Motion Path. Suppose the two endpoints of a line segment are B and C, taking the four control points as sequence as (B,C,B,C), then a straight line from B to C can be constructed as shown in Fig. 3. Analyze formula (7), when $\begin{cases}x_{i+2}=x_i\\ y_{i+2}=y_i\end{cases}$, $\begin{cases}x_{i+3}=x_{i+1}\\ y_{i+3}=y_{i+1}\end{cases}$, $\lambda=0$ and $k=-1$, formula (7) turns into the linear parameter equation:

$$\begin{cases}X=(x_i-x_{i+1})C^3+x_{i+1}\\ Y=(y_i-y_{i+1})C^3+y_{i+1}\end{cases}. \tag{8}$$

In formula: C is stand for $\cos\left(\pi u/2\right)$, $0\le u\le 1$.

Circular Arc Description of NC Motion Path. Using the spline curve by taking four special control points and shape adjustment coefficients can accurately describe the circular arc. Suppose the two endpoints of a circular arc segment are A and B, taking four continuous control points as $A(a,0)$, $B(0,a)$, $C(a,\pi a/2)$ and $D(-\pi a/2,a)$, shape adjustment coefficients as $\lambda=1$, $k=-1$, then a circular arc from A to B can be constructed. Put the above four control points' coordinates and shape adjustment coefficients into formula (7), the parametric equation of circular arc can be gotten:

$$\begin{cases}X=a\bullet C\\ Y=a\bullet S\end{cases}. \tag{9}$$

In formula: S is stand for $\sin\left(\pi u/2\right)$, C is stand for $\cos\left(\pi u/2\right)$, $0\le u\le 1$.

Fig. 4 shows the circular arc segment with center angle of 90°, which corresponding to the change of parameter u from 0 to 1. When using the spline curve to describe circular arc with center angle less than 90°, it can be done through the control of the value of parameter u; and when to describe circular arc with center angle more than 90°, using a few circular arc segments joining together to realize.

Free Curve Description of NC Motion Path. Using this cubic trigonometric interpolation spline can easily describe the free curve. From above we know that n given control points can construct $n-3$ segments of spline curves. As shown in Fig. 5, through (A, B, C, D, E, F, G, H, I, J) ten control points can generate a segment of free curve which formed by seven segments of small spline curves. And the whole free curve satisfies C^1 continuity, which ensures the smoothness of the spline curve.

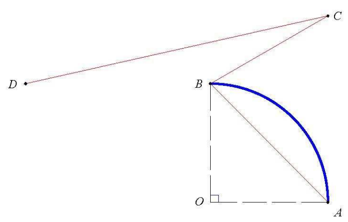


Fig. 4 Circular arc interpolation spline

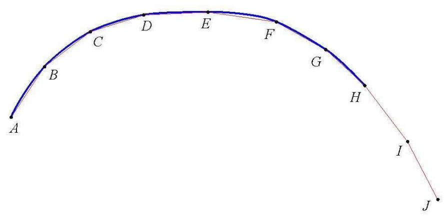


Fig. 5 Free curve interpolation spline

Planning of NC Motion Path. Fig. 6 shows a part of the current commonly used motion path in NC machining. The moving path of tool center is $A \to B \to C \to D$. During the motion path, points B and C are sharp turning points. When the corner angle formed by adjacent machining path segments is larger, it will cause oversized changes of movement velocity vector, which will be easier to cause larger impacts to NC machine tools. This phenomenon is especially more obvious under the circumstance of high-speed machining. According to the properties of the spline curve, inserting the spline at the corner of adjacent machining path segments, can generate the trajectories of corner transition and realize the smooth transfer between machining path segments, so as to achieve the purpose of smoothing corner transition speed.

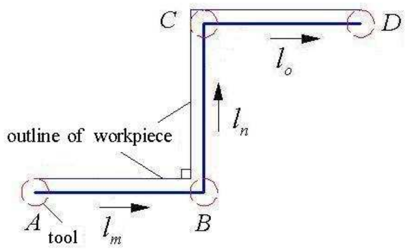


Fig. 6 NC motion path before smoothing

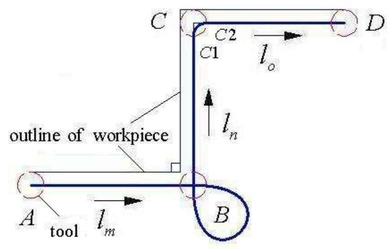


Fig. 7 NC motion path after smoothing

Fig. 7 shows the described NC motion path based on the adjustable sharp cubic trigonometric interpolation spline curve. Among them, points $C1$ and $C2$ are the intersections of tool outer contour line with segments BC and CD when the tool center is at point C. Segments AB, $BC1$, and $C2D$ are straight lines, insert a ring form spline transition curve outside the corner point B and circular arc spline transition curve inside the corner point C. Such planned NC machining motion path is smooth, and there is no speed drop and rise during the corner transition, which can guarantee the smooth transfer around the corner, and easy to realize the high-speed smooth machining between NC motion path segments.

Smoothing of External Corner. As shown in Fig. 8, in order to smooth transfer the machining segments between l_m and l_n, a ring form spline curve is inserted at the external corner point B, which is generated by the four control points with sequence as (B, B, $B1$, $B2$) and shape adjustment coefficient as $k = -1$. Among them, point $B1$ is on the extension cord of segment l_m and point $B2$ is on the segment l_n. Selecting the control points as this way can ensure that the starting point vector of the spline transfer curve is in the same direction with segment l_m, and the end point vector is in the same direction with segment l_n, which can realize the external corner smooth transition between NC motion path segments. Change the value of shape adjustment coefficient λ can generate different sizes of spline curves, so through choosing proper value of λ can control the change of acceleration during the corner movement process.

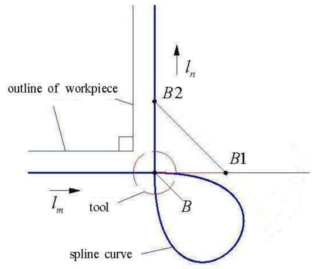


Fig. 8 External corner smooth transition

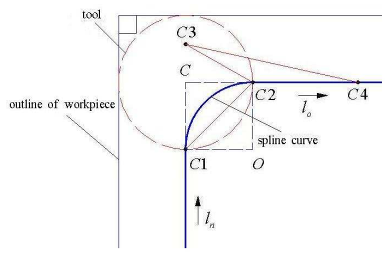


Fig. 9 Internal corner smooth transition

Smoothing of Internal Corner. As shown in Fig. 9, a circular arc spline curve is inserted at the internal corner point C to smooth transfer the segments between l_n and l_o. The four continuous control points are ($C1$, $C2$, $C3$, $C4$) and shape adjustment coefficient $k=-1$. Among them, points $C1$ and $C2$ are the intersections of tool outer contour line with segments l_n and l_o when the tool center is at point C; point $C3$ is on the extension cord of segment l_n, point $C4$ is on the segment l_o, and their coordinates are determined by points $C1$ and $C2$. Similarly, the starting point vector of this curve is in the same direction with segment l_n and the end point vector is in the same direction with segment l_o, which can realize the internal corner smooth transition of NC motion path segments. Change the value of shape adjustment coefficient λ can change the size of the transfer curves, so through choosing proper value of λ can control the change of acceleration during the corner movement process, meanwhile, it also can control the machining error.

Conclusion

This paper gives out a set of special basis function, and the constructed curve based on it is called cubic trigonometric interpolation spline curve which satisfies C^1 continuity. The curve has the following advantages: 1) Changing one control point only affects four segment curves which are related to it, and has no effects to other parts of the whole curve, so the curve has good property of locality. 2) Each segment of spline has two shape adjustment coefficients λ and k, and the shape of the curve can be easily controlled by changing the values of λ and k, so the curve has the property of shape adjustable. 3) Selecting proper control points and shape adjustment coefficients, the spline can precisely describe some common engineering curves such as straight line, circular arc and free curve, so the spline has the flexibility of curve structuring. 4) The spline using the form of polynomial expression to avoid the rational form, so it needs fewer amounts of calculation and storage space.

Therefore, using this spline to describe the complex NC motion paths has obvious advantages, and it also can be used to deal with the corner smooth transition between adjacent NC motion path segments. By inserting the adjustable sharp spline at the corner can easily realize the high-speed smooth transition between NC path segments, which can meet the needs of modern numerical control system for high-speed, stability and flexibility.

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