



# Empirical model based optimization of gearbox geometric design parameters to reduce rattle noise in an automotive transmission

Mehmet Bozca<sup>a,\*</sup>, Peter Fietkau<sup>b</sup>

<sup>a</sup> Yildiz Technical University, Mechanical Engineering Faculty, Machine Design Division, 34349, Yildiz, Istanbul, Turkey

<sup>b</sup> University of Stuttgart, Institute of Machine Element (IMA), D-70569, Pfaffenwaldring 9, Stuttgart, Germany

## ARTICLE INFO

### Article history:

Received 24 November 2009

Received in revised form 8 June 2010

Accepted 22 June 2010

Available online 8 August 2010

### Keywords:

Optimization

Gearbox

Rattle noise

Automotive transmission

## ABSTRACT

The optimization of gearbox geometric design parameters to reduce rattle noise in an automotive transmission based on an empirical model approach is studied. Rattle noise is calculated and simulated based on the design parameters of a 5-speed gearbox, and all pinion gears and wheel gears are helical. The effect of the design parameters on rattle noise is analyzed. The observed rattle noise profiles are obtained depending on the design parameters. During the optimization, an empirical average rattle noise level is considered as the objective functions and design parameters are optimized under several constraints that include bending stress, contact stress and a constant distance between gear centers. Therefore, by optimizing the geometric parameters of the gearbox such as, the module, number of teeth, axial clearance, and backlash, it is possible to obtain a light-weight-gearbox structure and minimize the rattling noise. It is concluded that the optimized geometric design parameters lower the rattle noise by 14% compared to the calculated rattle noise for sample gearbox. All optimized geometric design parameters also satisfy all constraints.

© 2010 Elsevier Ltd. All rights reserved.

## 1. Introduction

The optimization of gearbox geometric design parameters to reduce rattle noise in automotive transmissions based on an empirical model approach is studied.

Gear motion causes rattling and clattering noise, and noises level is considered to be a comfort factor in automotive industry. Therefore, reducing rattling and clattering noise in the gearbox is important in the automotive transmission for a comfortable car design.

Gears are widely used in automotive transmissions to transmit mechanical power from one shaft to another. The purpose of the gears is to couple two shafts together such that the rotation of the output, or driven, shaft is a function of the rotation of the input, or driving, shaft [4].

Rattling and clattering noise in automotive transmissions are caused by torsional vibration that is transmitted from the internal combustion engine to the transmission input shaft. This noise is known as rattling when the transmission is in neutral, and as clattering when the gear is engaged under power or in overrun [9–14].

Rattling and clattering are caused by torsional vibration of loose parts, i.e. parts, such as idler gears, synchronizer rings and sliding sleeves, which are not under load and therefore can move within their functional clearances [9,14].

Gear rattling noise is one of the major problems facing the industry, and the car industry in particular, because cars spend so much time idling under no load or very light loads [1].

The parameters that are responsible for rattle and clatter are classified as geometric parameters and operational parameter [9–14]. The geometric parameters include the module  $m$ , number of teeth  $z$ , helix angle  $\beta$ , axial clearance  $s_a$  and backlash  $s_v$ , as shown in Fig. 3. The operational parameters include the angular acceleration  $\hat{\omega}_1$  and excitation frequency  $\omega_{an}$  [9–14].

\* Corresponding author. Tel.: +90 212 383 27 93.

E-mail address: [mbozca@yildiz.edu.tr](mailto:mbozca@yildiz.edu.tr) (M. Bozca).

Transmission fluid as an engineering design parameter also has a considerable influence on rattling and clattering noises. The important factors include the type of oil, the additives used and the viscosity and the level of oil in the transmission which together act on a gear pair as drag torque, resulting in a reduction in rattling and clattering noises, especially at low speeds and when cold [9–14].

The analysis suggests that in order to reduce the gear rattle noise, rather than to increase the oil level in a gearbox and thus decreasing the mechanical efficiency, it would be opportune to guarantee the oil presence between the meshing teeth by means of a suitable lubrication device feeding the lubricant only in the meshing zone [15].

Sliding friction between meshing teeth is one of the primary excitations for noise and vibration in geared systems. Among the different kinds of non linearities in gear system, such as clearance, spatial variations and sliding friction, the effect of friction is the least understood. Certain unique characteristics of gear tooth sliding make it a potentially dominant factor. For instance, due to the reversal in direction during meshing action, friction is associated with a large oscillatory component, which causes a higher bandwidth in the system response. Furthermore, it becomes more significant at high values of torque and lower speeds, due to the tribological characteristics as well as due to higher force transmissibility in the sliding direction [17].

Optimization of the gear's macro-geometry i.e. the use of high contact ratio gears that has lead to minor noise emissions with higher transmitted power levels. Optimization of the gear's micro-geometry i.e. trying to balance load-induced teeth deflections with profile corrections that has generally lead to less noisy transmission effects. This is not a suitable solution for an overall working range; therefore profile corrections must be determined statistically to take into account manufacturing deviations which will overlap their effect [16].

### 1.1. Gearbox mechanism

The gearbox mechanism includes pinion gears, wheel gears, an input shaft, an output shaft, a lay shaft, a bearing support, and a synchronizers, as shown in Fig. 1.

#### 1.1.1. Pinion gears and gear wheels

All pinion gears and wheel gears are helical, and all gears are made of 16MnCr5.

#### 1.1.2. Input shaft

The constant pinion gear and rear wheel gear are engaged on the input shaft. The rear wheel gear is the idler gear and runs in the needle bearing on the input shaft. The synchronizer of the rear wheel gear is connected to the input shaft.

#### 1.1.3. Output shaft

The 1st, 2nd and 3rd wheel gears and 4th pinion gear are engaged on the output shaft. The 1st, 2nd and 3rd wheel gears are idler gears and run in the needle bearings on the output shaft. The synchronizer of the 1st and 2nd wheel gears is connected to the output shaft.

#### 1.1.4. Lay shaft

The 1st, 2nd, 3rd pinion gears, the rear pinion gear and the 4th wheel gear and the constant wheel gears, are engaged on the lay shaft. The 4th wheel gear is the idler gear and runs in the needle bearing on the lay shaft. The synchronizer of the 3rd and 4th wheel gears is connected to the lay shaft.

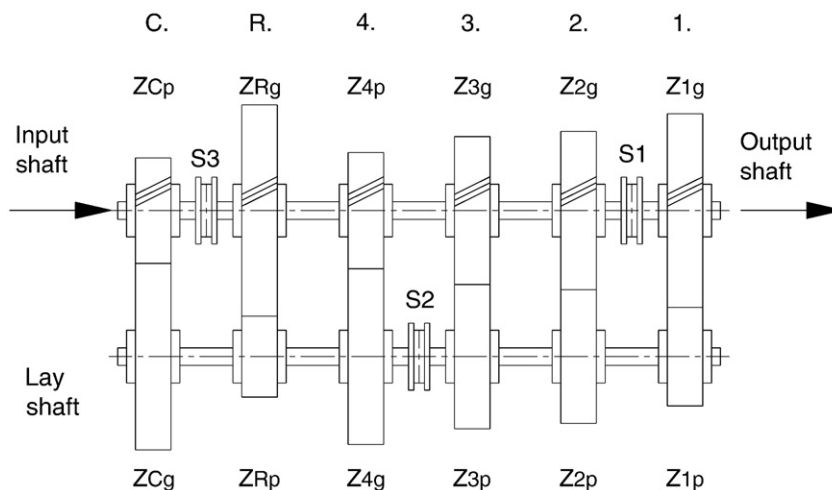


Fig. 1. 5-speed gearbox for automotive transmission.

### 1.1.5. Synchronizer

All synchronizers run in the needle bearings to maximize smoothness.

## 2. Calculation of rattle noise

Rattle noise is calculated and simulated based on the gearbox design parameters for a 5-speed gearbox for an automotive transmission, which is shown in Figs. 1 and 2.

The rattle noise level of a complete automotive transmission is calculated as follows [9–13]:

$$L_{pComp} = 10 \log \sum_{i=1}^n \left( 10^{0.1L_{p,i}} \right) \quad (1)$$

The rattle noise  $L_p$  is calculated as follows by correlating the computed noise value and the measured noise level [9–13]:

$$L_p = 10 \log(kI_m + 10^{0.1L_{basic}}) \quad (2)$$

where  $k$  is the calibration factor [–] and  $I_m$  is the average impact intensity [N]. The average impact intensity  $I_m$  is written as follows [9–13]:

$$I_m = m_2 \hat{\omega}_1 r_{b1} C_{lm} \quad (3)$$

where  $m_2$  is a loose part [kg],  $\hat{\omega}_1$  is the angular acceleration [rad/s<sup>2</sup>],  $r_{b1}$  is the pitch circle radius [mm] and  $C_{lm}$  is the related average impact intensity [–]. The average impact intensity,  $C_{lm}$  is written as follows [9–13]:

$$C_{lm} = \sqrt{C_{sv} \left( 1,462 - \frac{0,714C_{fa}C_{sa}}{-0,016C_{fa} + 0,12C_{sv}} \right)} \quad (4)$$

where  $C_{sv}$  is the non-dimensional circumferential backlash [–]. The non-dimensional circumferential backlash  $C_{sv}$  [–] is defined as follows [9–13]:

$$C_{sv} = \frac{s_v \omega_{an}^2}{r_{b1} \hat{\omega}_1} \quad (5)$$

where  $s_v$  is backlash [mm],  $\omega_{an}$  is the excitation frequency [rad/s] and  $C_{sa}$  is the non-dimensional axial clearance [–]. The non-dimensional axial clearance,  $C_{sa}$  [–] is defined as follows [9–13]:

$$C_{sa} = \frac{s_a \omega_{an}^2 \tan \beta}{r_{b1} \hat{\omega}_1} \quad (6)$$

where  $s_a$  is the axial clearance [mm],  $\beta$  is the helix angle [°],  $C_{fa}$  is the related axial friction force [–] and  $L_{basic}$  is the basic noise level [dB].



Fig. 2. View of 5-speed gearbox for automotive transmission.

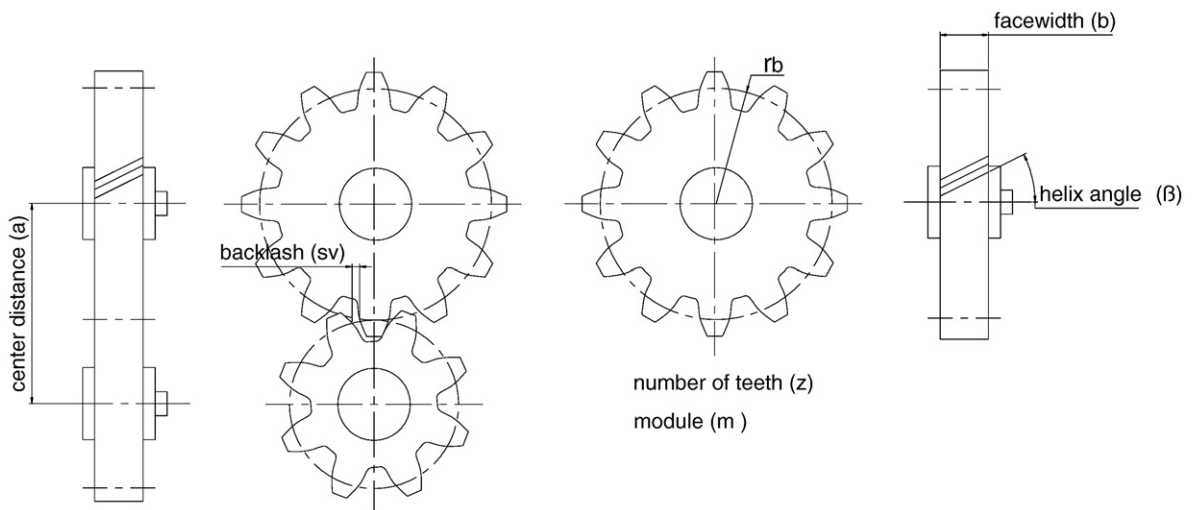


Fig. 3. Design parameters for a gearbox.

### 3. Effect of design parameters on rattle noise

The rattle noise is calculated and simulated based on the gearbox design parameters. Thus, the effect of design parameters on rattle noise is shown in Figs. 4–10 graphically. The gearbox design parameters are shown in Fig. 3. Rattle noise simulation parameters are shown in Table 1.

#### 3.1. Module ( $m$ )

As the module increases, rattle noise also increases. The module–rattle noise relationship is shown in Fig. 4.

#### 3.2. Number of teeth ( $z$ )

Increasing of number of teeth increases the rattle noise. Their relationship is shown in Fig. 5.

#### 3.3. Helix angle ( $\beta$ )

Changing the helix angle results in different levels of rattle noise. The relationship is shown in Fig. 6.

#### 3.4. Axial clearance ( $s_a$ )

Increasing of axial clearance causes the rattle noise to increase until axial clearance reaches its maximum value and decreases afterwards. The axial clearance–rattle noise relationship is shown in Fig. 7.

#### 3.5. Backlash ( $s_v$ )

Increasing the backlash result decreases the rattle noise until backlash reaches its maximum value; the rattle noise then increases. The backlash–rattle noise relationship is shown in Fig. 8.

#### 3.6. Angular acceleration ( $\hat{\omega}_1$ )

Increasing the angular acceleration also increases the rattle noise. The angular acceleration–rattle noise relationship is shown in Fig. 9.

#### 3.7. Excitation frequency ( $\omega_{an}$ )

Increasing the excitation frequency also increases the rattle noise. The excitation frequency–rattle noise relationship is shown in Fig. 10.

**Table 1**  
Rattle noise simulation parameters.

Parameters	Unit	1st pinion	2nd pinion	3rd pinion	4th pinion	Constant pinion	Rear pinion
Calibration factor $k$	[-]	1	1	1	1	1	1
Loose part $m_2$	[kg]	1	1	1	1	1	1
Angular acceleration $\hat{\omega}_1$	[rad/s <sup>2</sup> ]	500	500	500	500	500	500
Backlash $s_v$	[mm]	0.1	0.1	0.1	0.1	0.1	0.1
Excitation frequency $\omega_{an}$	[rad/s]	220	220	220	220	220	220
Axial clearance $s_a$	[mm]	0.3	0.3	0.3	0.3	0.3	0.3
Helix angle $\beta$	[mm]	26	30	30	30	30	30
Related axial friction force $C_{fa}$	[-]	0.5	0.5	0.5	0.5	0.5	0.5
Basic noise level $L_{basic}$	[dB]	65	65	65	65	65	65

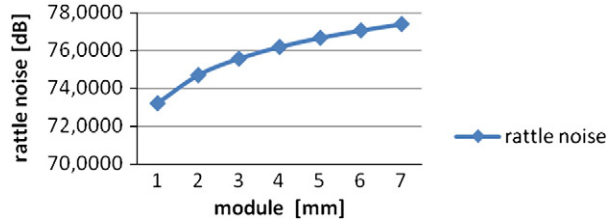


Fig. 4. Module-rattle noise relationship for the 1st gear.

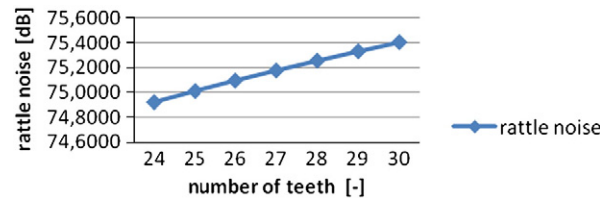


Fig. 5. Number of teeth-rattle noise relationship for the 1st gear.

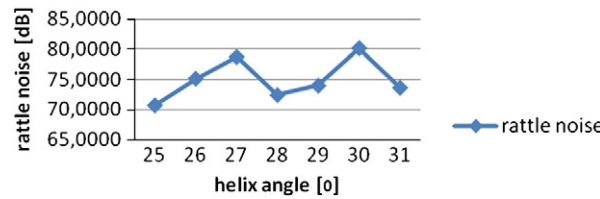


Fig. 6. Helix angle-rattle noise relationship for the 1st gear.

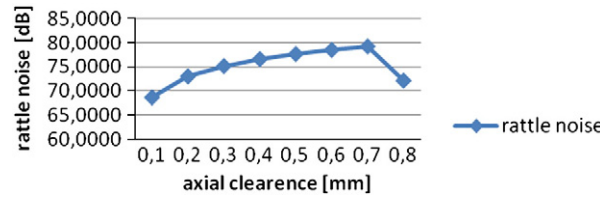


Fig. 7. Axial clearance-rattle noise relationship for the 1st gear.

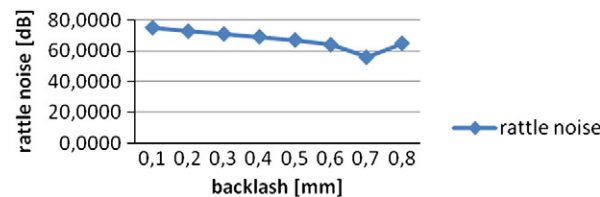


Fig. 8. Backlash-rattle noise relationship for the 1st gear.

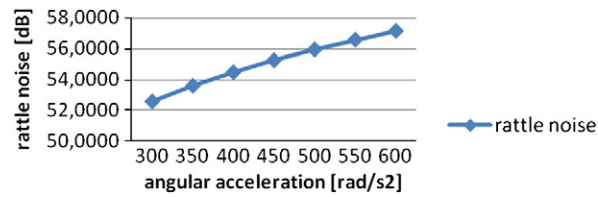


Fig. 9. Angular acceleration–rattle noise relationship for the 1st gear.

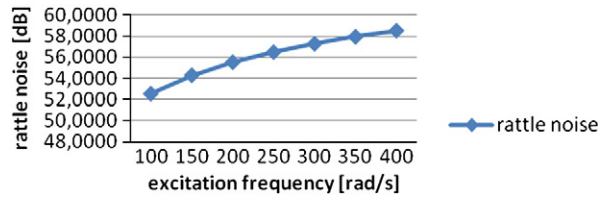


Fig. 10. Excitation frequency–rattle noise relationship for the 1st gear.

#### 4. Calculating the load capacity of helical gears

Determining the service life and/or determining the strength of gears are crucial for gear manufacturers. Gear strength is defined by the bending and contact strength [20].

##### 4.1. Tooth bending stress

The tooth bending stress is calculated as follows, Fig. 11 [2,3,5,7]. According to the ISO 6336, shear stresses due to lateral forces were not taken into account when determining the loading capacity of gear [3,20,21]. A tooth-root bending fatigue fracture usually starts at the 30° tangent of the root [3,8,21,22].

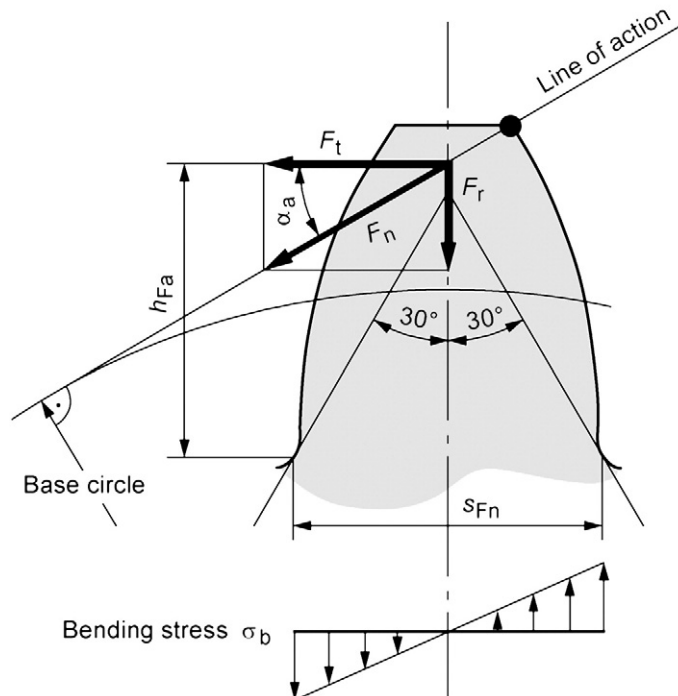


Fig. 11. Bending stress at the tooth-root.

The real tooth-root stress  $\sigma_F$ , is calculated as follows [2,3,5,7]:

$$\sigma_F = \frac{F_t}{b m_n} Y_F Y_S Y_\epsilon Y_\beta K_A K_V K_{F\beta} K_{F\alpha} \tag{7}$$

where  $F_t$  is the nominal tangential load [N],  $b$  is the face width [mm],  $m_n$  is the normal module [mm],  $Y_F$  is the form factor [-],  $Y_S$  is the stress correction factor [-],  $Y_\epsilon$  is the contact ratio factor [-],  $K_A$  is the application factor [-],  $K_V$  is the internal dynamic factor [-],  $K_{F\beta}$  is the face load factor for tooth-root stress [-] and  $K_{F\alpha}$  is the transverse load factor for tooth-root stress [-].

The permissible bending stress  $\sigma_{Fp}$  is calculated as follows [2,3,5,7]:

$$\sigma_{Fp} = \sigma_{Flim} Y_{ST} Y_N Y_\delta Y_R Y_X \tag{8}$$

where  $\sigma_{Flim}$  is the nominal stress number (bending) [N/mm<sup>2</sup>],  $Y_{ST}$  is the stress correction factor [-],  $Y_N$  is the life factor for the tooth-root stress [-],  $Y_\delta$  is the relative notch sensitivity factor [-],  $Y_R$  is the relative surface factor [-] and  $Y_X$  is the size factor that represents the tooth-root strength [-].

The safety factor for bending stress  $S_F$  is calculated as follows [2,3,5,7]:

$$S_F = \frac{\sigma_{Fp}}{\sigma_F} \tag{9}$$

#### 4.2. Tooth contact stress

The tooth contact stress  $\sigma_{HC}$  is calculated as follows, as seen in Fig. 12 [2,3,5,6]:

The real contact stress  $\sigma_H$  is calculated as follows [2,3,5,6]:

$$\sigma_H = \sqrt{\frac{F_t}{b m_n} \frac{u + 1}{u}} Z_H Z_E Z_\epsilon Z_\beta \sqrt{K_A K_V K_{H\beta} K_{H\alpha}} \tag{10}$$

where  $d_1$  is the reference diameter of the pinion [mm],  $u$  is gear ratio [-],  $Z_H$  is the zone factor [-],  $Z_E$  is the elasticity factor [ $\sqrt{N/mm^2}$ ],  $Z_\epsilon$  is the contact ratio factor [-],  $Z_\beta$  is the helix angle factor [-],  $K_{H\beta}$  is the face load factor for contact stress [-] and  $K_{H\alpha}$  is the transverse load factor for contact stress [-].

The permissible contact stress  $\sigma_{Hp}$  is calculated as follows [2,3,5,6]:

$$\sigma_{Hp} = \sigma_{Hlim} Z_N Z_L Z_V Z_R Z_W Z_X \tag{11}$$

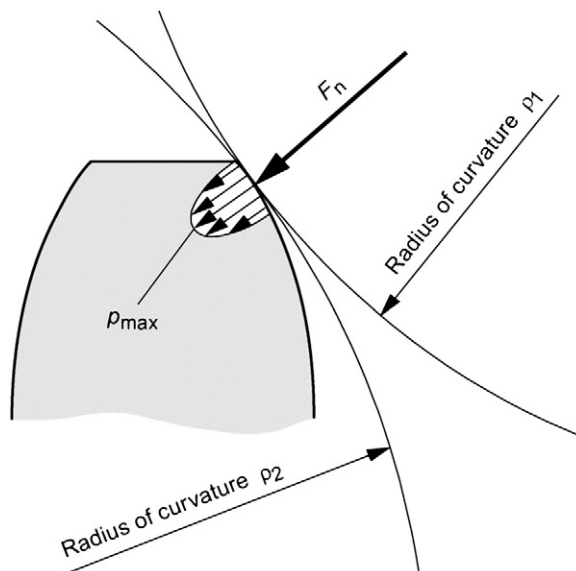


Fig. 12. Contact stress at the tooth flank.

where  $\sigma_{Hlim}$  is the allowable stress numbers (contact) [N/mm<sup>2</sup>],  $Z_N$  is the life factor for contact stress [-],  $Z_L$  is the lubrication factor [-],  $Z_V$  is the velocity factor [-],  $Z_R$  is the roughness factor [-],  $Z_W$  is the work hardening factor [-] and  $Z_X$  is the size factor for contact stress [-].

The safety factor for contact stress  $S_H$  is calculated as follows [2,3,5,6]:

$$S_H = \frac{\sigma_{Hp}}{\sigma_H} \quad (12)$$

## 5. Optimization of gearbox design parameters

Constrained optimization is a very useful tool for light-weight-structure design of machine elements with constraints such as stress, deformation and vibration.

In optimization, the goal is usually to minimize the cost of a structure while satisfying the design specifications [19]. By optimizing the responsible parameters, it is possible to obtain a light-weight-gearbox structure and minimize the rattling noise [13].

Let  $F(X)$  denote the objective function to be minimized, where  $\mathbf{X}$  is the design parameter (variable) vector to be determined. Then, to find the constrained minimum of  $F(X)$ , the following optimization problem is solved [18,19]:

$$\min F(X) \quad (13)$$

$$\text{subject to : } \mathbf{LB} \leq \mathbf{X} \leq \mathbf{UB} \text{ and } G(X) \leq 0 \quad (14)$$

where,  $\mathbf{LB}$  and  $\mathbf{UB}$  define, the sets of lower and upper bounds on the design parameter (variable)  $\mathbf{X}$ . Iterations start with the initial design parameter vector  $\mathbf{X}_0$  and a solution vector  $\mathbf{X}$  is found that minimizes the objective function  $F(X)$  subject to the nonlinear inequalities  $G(X) \leq 0$  [18,19].

## 6. Numerical example

Constrained optimization approaches are applied to the 5-speed gearbox for automotive transmission. All programs are developed using the MATLAB program. In all optimization studies, the sequential quadratic programming (SQP) method is employed.

To find the optimum design parameter, the initial design parameters of the 5-speed gearbox for automotive transmission such as  $m$ ,  $z$ ,  $\beta$ ,  $b$ ,  $s_a$ , and  $s_v$  are varied. Thirty-six design parameters are optimized simultaneously using the developed programs. During optimization, different initial value vectors are used to identify the global minimum solution of the objective function  $L_{p_{average}}(m, z, \beta, b, s_a, s_v)$ .

### 6.1. Objectives function

Average of rattle noise levels of the gear system are considered as objective functions and the design parameters are optimized considering bending stress, contact stress and distance between gear center constraints. The flowchart of the design parameter optimization procedure is shown in Fig. 13.

The following objective function is employed:

$$F = L_{p_{average}} \quad (15)$$

Average of rattle noise levels  $L_{p_{average}}$  are defined as follows:

$$L_{p_{average}} = \frac{1}{n} \sum_{i=1}^n L_{p_{Complete.i}} \quad (16)$$

Average of rattle noise levels  $L_{p_{average}}(m, z, \beta, b, s_a, s_v)$  are considered to be the objective functions to be minimized, where module  $m$ , the number of teeth  $z$ , helix angle  $\beta$ , axial clearance  $s_a$  and backlash  $s_v$  are the design parameters (variables) to be determined. Then, to find the constrained minimum of the average of rattle noise level  $L_{p_{average}}(m, z, \beta, b, s_a, s_v)$ , the following optimization problem is solved:

$$\min L_{p_{average}}(m, z, \beta, b, s_a, s_v) \quad (17)$$

$$\text{subject to : } \mathbf{LB} \leq m, z, \beta, b, s_a, s_v \leq \mathbf{UB} \text{ and } G(X) \leq 0 \quad (18)$$

where,  $\mathbf{LB}$  and  $\mathbf{UB}$  define, the sets of lower and upper bounds on the design parameters (variables) vector such as  $m$ ,  $z$ ,  $\beta$ ,  $b$ ,  $s_a$ , and  $s_v$ . Iteration begins with the initial design parameter vector, which include, e.g.,  $m_0$ ,  $z_0$ ,  $\beta_0$ ,  $b_0$ ,  $s_{a0}$  and  $s_{v0}$  and a solution vector



with  $m, z, \beta, b, s_a,$  and  $s_v$  is found that minimizes the objective function  $L_{p_{average}}(m, z, \beta, b, s_a, s_v)$  subject to the nonlinear inequalities  $G(X) \leq 0$ .

6.2. Constraint functions

Tooth bending stress, contact stress and distance between gear centers are considered to be the constraint functions in the optimization. The tooth bending stress parameters and tooth contact stress parameters are shown in Tables 2 and 3 respectively. The following constraints are considered to be constraint functions

$$1. \sigma_F - \sigma_{Fp} \leq 0 \tag{19}$$

where  $\sigma_F$  is the real tooth-root stress [N/mm<sup>2</sup>] and  $\sigma_{Fp}$  is the permissible bending stress [N/mm<sup>2</sup>].

$$2. \sigma_H - \sigma_{Hp} \leq 0 \tag{20}$$

where  $\sigma_H$  is the real contact stress [N/mm<sup>2</sup>] and  $\sigma_{Hp}$  is the permissible contact stress [N/mm<sup>2</sup>].

$$3. a_1 = a_2 = a_3 = a_4 = a_5 = a_R = \text{constant} \tag{21}$$

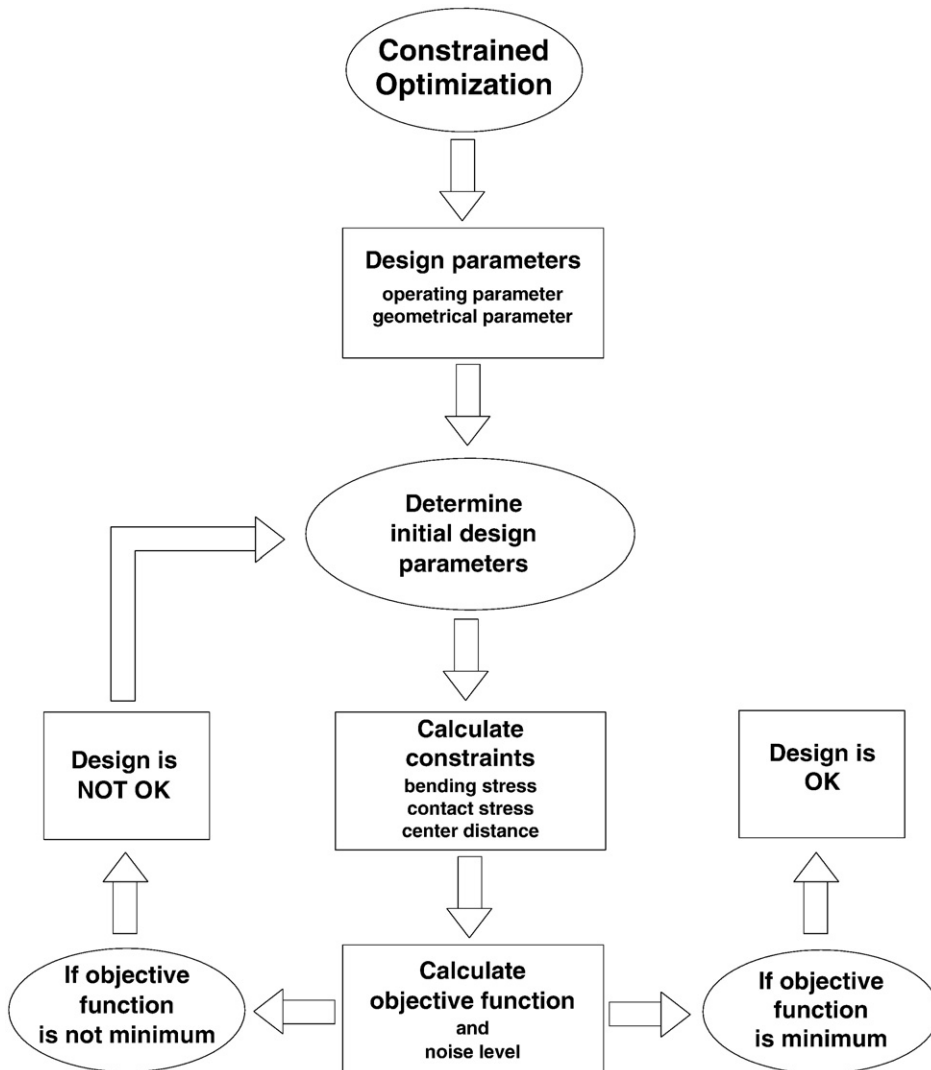


Fig. 13. Flow chart to optimize gearbox design parameters.

**Table 2**  
Tooth bending stress parameters.

Parameter	Unit	1st pinion	2nd pinion	3rd pinion	4th pinion	Constant pinion	Rear pinion
Torque $T_L$	[N.mm]	$392 \cdot 10^3$	$392 \cdot 10^3$	$316 \cdot 10^3$	$252 \cdot 10^3$	$200 \cdot 10^3$	$1148 \cdot 10^3$
Gear ratio $u$	[-]	1.814	1.147	1.242	1.560	1	2.84
Stress correction factor $Y_{ST}$	[-]	2	2	2	2	2	2
Form factor $Y_F$	[-]	2.75	2.75	2.75	2.75	2.75	2.75
Stress correction factor $Y_S$	[-]	1.60	1.60	1.60	1.60	1.60	1.60
Transverse contact ratio $\varepsilon_\alpha$	[-]	1	1	1	1	1	1
Overlap ratio $\varepsilon_\beta$	[-]	1	1	1	1	1	1
Application factor $K_A$	[-]	1.25	1.25	1.25	1.25	1.25	1.25
Internal dynamic factor $K_V$	[-]	1.14	1.14	1.14	1.14	1.14	1.14
Transverse load factor for tooth-root stress $K_{F\alpha}$	[-]	1.2	1.2	1.2	1.2	1.2	1.2
Nominal stress number (bending) $\sigma_{Hlim}$	[N/mm <sup>2</sup> ]	300	300	300	300	300	300
Life factor for tooth-root stress $Y_N$	[-]	1	1	1	1	1	1
Relative notch sensitivity factor $Y_\delta$	[-]	1	1	1	1	1	1
Relative surface factor $Y_R$	[-]	1	1	1	1	1	1
Size factor relevant to tooth-root strength $Y_X$	[-]	1	1	1	1	1	1

where  $a_1$  is the center distance of the 1st speed,  $a_2$  is the center distance of the 2nd speed,  $a_3$  is the center distance of the 3rd speed,  $a_4$  is the center distance of the 4th speed,  $a_5$  is the center distance of the 5th speed and  $a_R$  is the center distance of rear speed.

### 6.3. Optimization results

The optimization results using objective function  $F$  are presented in Table 4. Because of the limited space, only the important results are presented.

It is observed in solution 1 that the obtained optimum module changes between 3.2895 [mm] and 4.3290 [mm]. The optimum number of teeth varies between 14 [-] and 19 [-]. In addition, the optimum helix angle varies between 24.8929 [°] and 26.5295 [°] while optimum face width varies between 27 [mm] and 28 [mm]. The optimum axial clearance varies between 0.2000 [mm] and 0.5000 [mm] while the optimum backlash varies between 0.2000 [mm] and 0.4952 [mm]. The safety factor for bending stress  $S_F$  ranges between 1.000 and 2.2651. In addition, the safety factor for contact stress  $S_H$  varies between 1.1627 and 1.8533. The rattle noise values of the optimized gearbox change between 72 [dB] and 78 [dB].

It is observed in solution 2 that the optimum module ranges from 3.1063 [mm] and 4.3290 [mm]. The numbers of teeth vary between 14 [-] and 19 [-]. Moreover, the optimum helix angle varies between 30.1647 [°] and 31.9655 [°] and the optimum face width varies between 30 [mm] and 32 [mm]. The optimum axial clearance varies between 0.2085 [mm] and 0.6000 [mm] while the optimum backlash varies between 0.2000 [mm] and 0.3583 [mm]. The safety factor for bending stress  $S_F$  ranges between 1.000 and 2.6888. In addition, the safety factor for contact stress  $S_H$  varies between 1.1542 and 1.9563. The rattle noise values of the optimized gearbox change between 73 [dB] and 78 [dB].

Although the results given above represent the optimum solution, standard design parameter values used by gear manufacturers do not necessarily reflect these results because some of the solutions are impossible in practice.

The rattle noise values vary between 72 [dB] and 78 [dB] during optimization. Thus, by optimizing the design parameters, the rattle noise values are reduced to 72 [dB]. These rattle noise values are between 10% and 14% lower than the calculated rattle noise values for the sample gearbox.

The safety factor for bending stress  $S_F$  ranges between 1.00 and 2.68 during optimization. In addition, the safety factor for contact stress  $S_H$  varies between 1.15 and 1.95 during optimization. Thus, all optimized design parameters satisfy all constraints.

The CPU time varies between 9 [s] and 13 [s] using objective function  $F$  during optimization.

**Table 3**  
Tooth contact stress parameters.

Parameter	Unit	1st pinion	2nd pinion	3rd pinion	4th pinion	Constant pinion	Rear pinion
Reference diameter of pinion $d_1$	[mm]	61.116	80.124	76.716	67.199	58.151	41.319
Gear ratio $u$	[-]	1.814	1.147	1.242	1.560	1	2.84
Zone factor $Z_H$	[-]	1	1	1	1	1	1
Elasticity factor $Z_E$	$[\sqrt{N/mm^2}]$	189.8	189.8	189.8	189.8	189.8	189.8
Transverse load factor for contact stress $K_{H\alpha}$	[-]	1.2	1.2	1.2	1.2	1.2	1.2
Allowable stress numbers (contact) $\sigma_{Hlim}$	[N/mm <sup>2</sup> ]	800	800	800	800	800	800
Life factor for contact stress $Z_N$	[-]	1	1	1	1	1	1
Velocity factor $Z_V$	[-]	1	1	1	1	1	1
Roughness factor $Z_R$	[-]	1	1	1	1	1	1
Work hardening factor $Z_W$	[-]	1	1	1	1	1	1
Size factor for contact stress $Z_X$	[-]	1	1	1	1	1	1

**Table 4**Optimization results by using objective function  $F$ .

No. 1						
Lb = [2 2 2 2 2 14 14 14 14 14 14 20 20 20 20 20 20 20 20 20 20 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2];						
Ub = [6 6 6 6 6 24 24 24 24 24 24 32 32 32 32 32 28 28 28 28 28 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5];						
x0 = [5 5 5 5 5 18 18 18 18 18 18 26 26 26 26 26 28 28 28 28 28 0.4 0.4 0.4 0.4 0.4 0.4 0.4 0.4 0.4 0.4];						
Solution						
	1st pinion	2nd pinion	3rd pinion	4th pinion	Constant	Rear
$m$	4.0508	3.9149	3.6147	3.2895	4.1657	4.3290
$z$	14.0000	19.0000	19.0000	19.0000	19.0000	14.0000
$\beta$	25.8290	25.6618	25.6777	24.8929	25.5629	26.5295
$b$	27.9989	27.9998	27.9802	28.0000	28.0000	28.0000
$s_a$	0.4771	0.4885	0.4929	0.2000	0.3763	0.5000
$s_v$	0.3474	0.4952	0.4834	0.2000	0.4945	0.2000
$S_F$	1.0073	1.0888	1.2092	1.0000	2.2651	1.0000
$S_H$	1.1627	1.3079	1.4268	1.3774	1.8533	1.4951
$a$	79.7923	79.8507	76.9895	80.0000	79.1424	80.0000
$L_p$	73.1590	51.3262	44.5129	63.7672	68.9288	77.1255
$L_{pcomp}$	78.0791	78.0791	78.0791	76.7788	78.0791	72.2078
$L_{paverage}$	76.8838					
CPU	13.6376					
No. 2						
Lb = [2 2 2 2 2 14 14 14 14 14 14 20 20 20 20 20 24 24 24 24 24 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2];						
Ub = [7 7 7 7 7 28 28 28 28 28 28 32 32 32 32 32 32 32 32 32 32 0.6 0.6 0.6 0.6 0.6 0.6 0.6 0.6 0.6 0.6];						
x0 = [7 7 7 7 7 21 21 21 21 21 21 31 31 31 31 31 30 30 30 30 30 0.6 0.6 0.6 0.6 0.6 0.6 0.6 0.6 0.6 0.6];						
Solution						
	1st pinion	2nd pinion	3rd pinion	4th pinion	Constant	Rear
$m$	4.0613	3.7761	3.5957	3.1063	4.0726	4.3290
$z$	14.0000	19.0000	19.0000	19.0000	19.0000	14.0000
$\beta$	30.9423	31.9655	31.9336	31.9218	31.9235	30.1647
$b$	32.0000	30.0093	30.0000	30.0000	30.0000	32.0000
$s_a$	0.2085	0.2402	0.2097	0.2177	0.3261	0.6000
$s_v$	0.2000	0.2494	0.2391	0.2582	0.3583	0.2000
$S_F$	1.0000	1.2268	1.3511	1.2699	2.6888	1.0000
$S_H$	1.1542	1.3624	1.4656	1.5025	1.9563	1.1867
$a$	80.0000	77.0184	76.5839	75.5449	77.3796	80.0000
$L_p$	73.1550	45.8808	57.1443	42.3357	52.8875	76.9832
$L_{pcomp}$	76.9832	78.4881	78.4881	78.4881	78.4881	73.1547
$L_{paverage}$	77.3484					
CPU	9.7441					

## 7. Comparison of rattle noise levels

The sample gearbox rattle noise level and optimized gearbox rattle noise levels using objective function  $F$  for the 1st, 2nd, 3rd, 4th, 5th and rear speed are shown below.

A comparison of the rattle noise level of the sample gearbox and the optimized gearbox for the 1st speed is shown in Fig. 14. While the rattle noise of the sample gearbox for the 1st speed is 88.1524 [dB], the rattle noise of the optimized gearbox for 1st speed is 78.0791 [dB].

A comparison of the rattle noise level of the sample gearbox and the optimized gearbox for the 2nd speed is shown in Fig. 15. While rattle noise of the sample gearbox for the 2nd speed is 88.1893 [dB], rattle noise of the optimized gearbox for the 2nd speed is 78.0791 [dB].

A comparison of the rattle noise level of the sample gearbox and the optimized gearbox for the 3rd speed is shown in Fig. 16. While the rattle noise of the sample gearbox for the 3rd speed is 86.3327 [dB], the rattle noise of the optimized gearbox for the 3rd speed is 78.0791 [dB].

A comparison of the rattle noise level of the sample gearbox and the optimized gearbox for the 4th speed is shown in Fig. 17. While rattle noise of the sample gearbox for the 4th speed is 86.3327 [dB], the rattle noise of the optimized gearbox for the 4th speed is 76.7788 [dB].

A comparison of the rattle noise level of the sample gearbox and the optimized gearbox for the 5th speed is shown in Fig. 18. While the rattle noise of the sample gearbox for the 4th speed is 88.4915 [dB], the rattle noise of the optimized gearbox for the 5th speed is 78.0791 [dB].

The rattle noise level of the sample gearbox and the optimized gearbox for the rear speed is compared in Fig. 19. While the rattle noise of the sample gearbox for the rear speed is 87.7589 [dB], the rattle noise of the optimized gearbox for the rear speed is 72.2078 [dB].

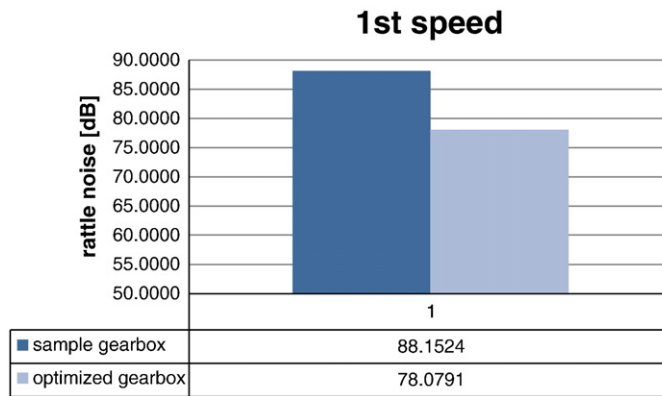


Fig. 14. Comparison between the rattle noise level of the sample gearbox and the optimized gearbox for the 1st speed.

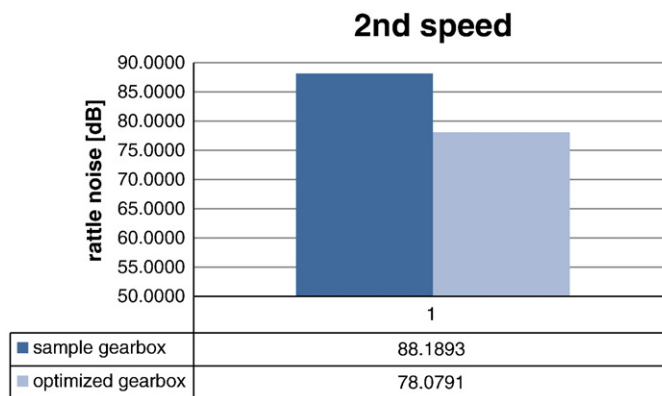


Fig. 15. Comparison between the rattle noise level of the sample gearbox and the optimized gearbox for the 2nd speed.

It is shown that the rattle noise values of the optimized gearbox are lower than the calculated rattle noise values for the sample gearbox for each speed.

## 8. Conclusion

It is concluded through simulation that increasing geometric parameters of the gearbox, such as the module and number of teeth results in increased rattle noise. In addition, increased axial clearance results in increased rattle noise until the axial clearance reaches its maximum value, then the rattle noise decreases. Moreover, increased backlash causes decreased rattle noise until the backlash reaches its maximum value, then the rattle noise increases. Changing the helix angle resulted in different levels of rattle

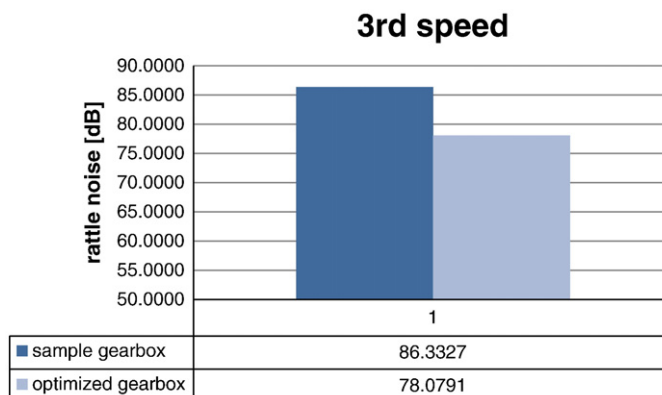


Fig. 16. Comparison between the rattle noise level of the sample gearbox and the optimized gearbox for the 3rd speed.

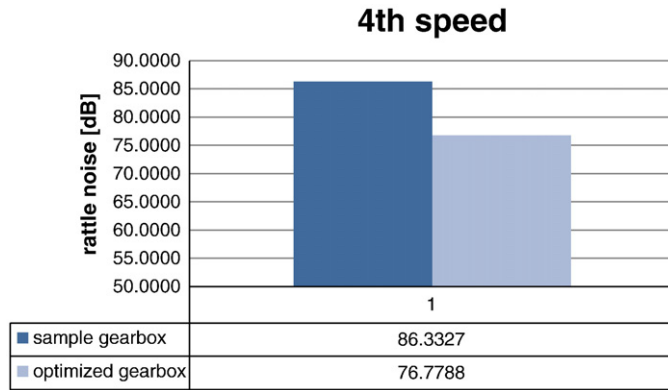


Fig. 17. Comparison between the rattle noise level of the sample gearbox and the optimized gearbox for the 4th speed.

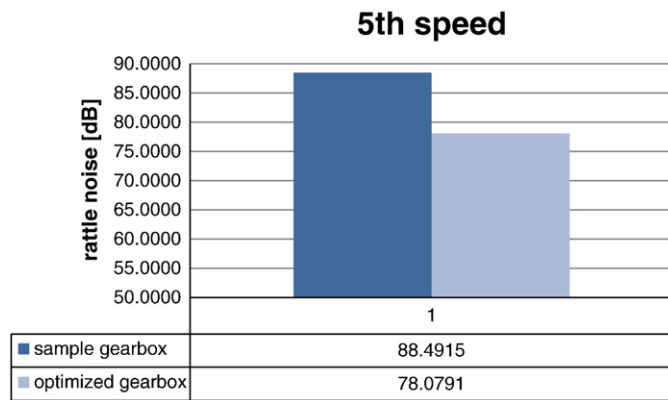


Fig. 18. Comparison between the rattle noise level of the sample gearbox and the optimized gearbox for the 5th speed.

noise, while changing of the face width resulted in a constant level of rattle noise. Furthermore, increasing the gearbox operational parameters, such as the angular acceleration and excitation frequency, caused increased gearbox rattle noise.

Some geometric design parameters, such as the module, and number of teeth must satisfy desired safety protocols, and some backlash is necessary to allow room for an oil film for all conditions of thermal expansion and contraction. Although, there is no relationship between face width and rattle noise, face width is necessary to satisfy the desired contact safety requirement. Therefore, by optimizing geometric parameters of the gearbox including, the module, number of teeth, axial clearance, and backlash, it is possible to obtain a light-weight-gearbox structure and minimize the rattling noise.

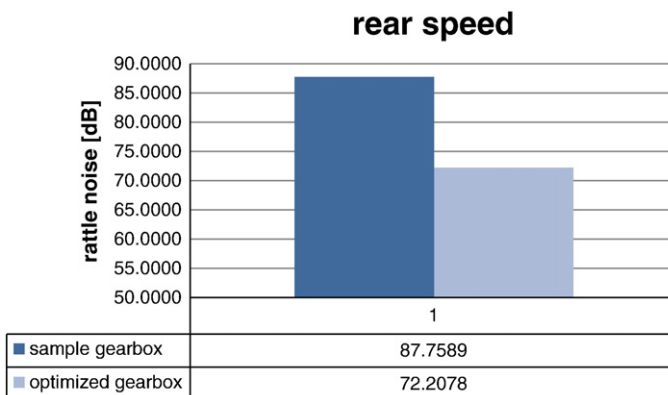


Fig. 19. Comparison between the rattle noise level of the sample gearbox and the optimized gearbox for the rear speed.

Optimized geometric design parameters lower the rattle noise by 14% compared to the calculated rattle noise values for the sample gearbox. All optimized geometric design parameters also satisfy all constraints. Optimizing the geometric design parameters not only reduces the rattle noise but also increases the desirable bending stress and contact stress level.

While geometric parameters, such as the module, number of teeth, helix angle, face width, backlash and axial clearance are optimized, the operational parameters such as angular acceleration and excitation frequency are not optimized because these operational parameters are given by the automotive manufacturer as input values.

## Acknowledgements

This research study is supported by Institute of Machine Elements (IMA), University of Stuttgart. Authors thank Prof. Dr.-Ing. B. Bertsche, Dipl.-Ing. A. Baumann, Dipl.-Ing. W. Novak, Dipl.-Ing. S. Nebel and Dipl.-Ing. S. Sanzenbacher for their helpful co-operation.

## References

- [1] J.D. Smith, *Gear Noise and Vibration*, Second Edition, Markel Dekker, Inc, Newyork, 2003.
- [2] H. Naunheimer, B. Bertsche, G. Lechner, *Fahrzeuggetriebe*, 2. Auflage, Springer Verlag, Berlin Heidelberg, 2007.
- [3] Roloff/Matek, *Maschinenelemente*, Vieweg & Sohn Verlag/Fachverlage GmbH, Wiesbaden, 2005.
- [4] H.D. Eckhardt, *Kinematic Design of Machines and Mechanisms*, McGraw-Hill, U.S.A, 1998.
- [5] ISO 6336-1: Calculation of load capacity of spur and helical gears, part 1: Basic principle, introduction and general influence factors.
- [6] ISO 6336-2: Calculation of load capacity of spur and helical gears, part 2: Calculation of surface durability (pitting).
- [7] ISO 6336-3: Calculation of load capacity of spur and helical gears, part 3: Calculation of tooth bending strength.
- [8] ISO 6336-5: Calculation of load capacity of spur and helical gears, part 5: Strength and quality of materials.
- [9] S.N. Dogan, J. Ryborz, B. Bertsche, Design of low-noise manual automotive transmissions, Proc. IMechE 220 Part K: J. Multi-body Dynamics (2006) 79–95.
- [10] S.N. Dogan, J. Ryborz, B. Bertsche, Rattling and clattering noise in automotive transmission-simulation of drag torque and noise.
- [11] B. Bertsche, W. Nowak, M. Stockmeier, S.N. Dogan, J. Ryborz, Development process of clatter and rattle noise free (CARF-) transmissions, Proceedings of IDETC/CIE 2005, ASME, Long Beach, California, USA, September 2005, pp. 1–9.
- [12] S.N. Dogan, J. Ryborz, B. Bertsche, Low-noise automotive transmission investigations of rattling and clattering, 3rd International Symposium on Multi-body Dynamics: Monitoring & Simulation techniques, Loughborough University, Loughborough, Leicestershire, UK, July 2004, pp. 1–18.
- [13] S.N. Dogan, Loose part vibration in vehicle transmission-gear rattle, Turkish Journal of Engineering and Environmental Science, (TÜBITAK), 23 (1999) 439–454.
- [14] W. Novak, M. Stockmeier, B. Bertsche, Improvement of power train efficiency via CARF-Transmissions, International Conference on Engineering Design, ICED 2005, Melbourne, August 2005.
- [15] R. Brancati, E. Rocca, R. Russo, A gear rattle model accounting for oil squeeze between the meshing gear teeth, Proc. IMechE 219 Part D: J. Automobile Engineering (2005) 1075–1083.
- [16] P. Bellomo, F. Cricenti, N. De Vito, C.H. Lang, D. Minervini, Innovative vehicle power train system engineering: beating the noisy offenders in vehicle transmissions, Society of Automotive Engineers, ASE 2000 World Congress, Detroit, Michigan, USA, March 2000.
- [17] M. Vaishya, R. Singh, Strategies for modeling friction in gear dynamics, Journal of Mechanical Design (2003) 383–393.
- [18] M. Bozca, Integrated optimum design of structures and controllers, Ph.D. dissertation submitted to Istanbul Technical University, Istanbul, Turkey, 2005.
- [19] M. Bozca, A. Mugan, H. Temeltas, Decoupled approach to integrated optimum design of structures and robust control systems, Structural and Multidisciplinary Optimization, Springer 36 (2007) 169–191.
- [20] M. Bozca, Influence of core hardness on bending strength of tooth in cylindrical gears under quasistatic loading conditions, Fatigue & Fracture of Engineering Materials & Structures, Blackwell Publishing Ltd., U.K. 31 (2008) 902–910.
- [21] G.P. Cavallaro, T.P. Wilks, C. Subramanian, C. Straffordt, K.N.P. French, J.E. Allison, Bending fatigue and contact fatigue characteristics of carburized gears, Surf. Coat. Technol. 71 (1995) 182–192.
- [22] M. Mackaldener, M. Olsson, Interior fatigue of gear teeth, Fatigue & Fracture of Engineering Materials & Structures 23 (2000) 283–292.